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Diagnostics of the railway friction disc brake based on the analysis of the vibration signals in terms of resonant frequency

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Abstract Increasing the speed of trains forces the application of a greater braking power, i.e. the capability of the brake system of rapidly absorbing and dispersing the stored heat energy. The instability of operation results from the occurrence of vibrations on the friction component, which affects the efficiency of the braking process. In practice, it means that when braking, the vehicle variable friction resistance may lead to an uneven braking process. The effects of such changes can be manifested in the form of vibrations with high input amplitude. The vibrations generated by the working components are distributed throughout the vehicle, which also adversely affects the driving comfort. The purpose of this paper is to determine the resonant frequencies of selected components of the railway disc brake. This paper presents a method of assessing the thickness of the brake pads of a railway disc brake based on the results of parameterization of the time-frequency map $a(t, f)$ of vibrations generated by the brake components. Methods of time-frequency analysis were suggested as a tool of time-frequency selection of the vibration signal. The authors described an algorithm of the method and presented an example illustrating the application of the synchronously averaged short-time Fourier transform for the extraction of the signal components related to the braking process. As a result of parameterization of the map $a(t, f)$ signal parameters, values covariant with the thickness of the disc brake were obtained.

Keywords Railway · Coach · Disc brake · Vibroacoustic diagnostics · Short-time Fourier transform

1 Introduction

A part of the paper was presented on the 13th International Conference—Dynamic Systems Theory and Applications, December 7–10, 2015 Łódź, Poland [1]. The paper has been extended by problems concerning the testing of the brake system of railway coaches under operation. Possibilities of application of different methods for the estimation of the value of natural frequency, depending on the type of diagnostic test (when stationary or driving) were indicated.

One of the negative factors affecting the human environment (especially the acoustic one) is the noise generated by motor vehicles.

Unfortunately, vibroacoustic phenomena generated particularly by brake systems are regarded as highly undesirable. The problem of vibration in the brakes of both rail and road vehicles was already indicated in the 1930s of the last century [2–4]. At the beginning, the researchers pointed out that the vibration and noise generated by brakes, originate in the friction characteristics of the tribological pair, conditioning the occurrence of this effect on the fact that the coefficient of friction changes its value with the rotational speed. It was also claimed that the coefficient of static friction is greater than that of kinetic friction. The consequence of vibration

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in the brake, in combination with the propagation of sound, is the instability of the system operation, which is manifested by a variable course of the instantaneous coefficient of friction.

2 Selected models of vibration in the brakes

The basic components of a disc brake system are: a brake disc rigidly connected with the wheelset and the calliper (holder) with a friction pad fixed in it, as shown in Fig. 1. For the calculation of the disc lever system parameters, models with varying degrees of simplification are used. Due to the application of models of a disc brake lever system, they can be classified as follows:

- models for kinematic calculations,
- models for calculations of the braking parameters,
- models for calculations of the natural frequency of the lever mechanism components.

In papers [5,6] in order to calculate the kinematic quantities (the displacement of individual components of the brake lever system), a structural disc brake lever system has been proposed.

The movement of the system shown in Fig. 2a has been described by a system of differential equations.

$$\begin{cases} m_1 \ddot{x}_1 + c_{1,x}(\dot{x}_1 - \dot{x}_0) + k_{1,x}(x_1 - x_0) + c_{2,x}(\dot{x}_1 - \dot{x}_2) + k_{2,x}(x_1 - x_2) = F_B \\ m_2 \ddot{x}_2 + c_{2,x}(\dot{x}_2 - \dot{x}_1) + k_{2,x}(x_2 - x_1) = 0 \\ m_1 \ddot{y}_1 + c_{1,y}(\dot{y}_1 - \dot{y}_0) + k_{1,y}(y_1 - y_0) + c_{2,y}(\dot{y}_1 - \dot{y}_2) + k_{2,y}(y_1 - y_2) = N \\ m_2 \ddot{y}_2 + c_{2,y}(\dot{y}_2 - \dot{y}_1) + k_{2,y}(y_2 - y_1) + c_{3,y}(\dot{y}_2 - \dot{y}_0) + k_{3,y}(y_2 - y_0) = 0 \end{cases} \quad (1)$$

where m_1 , friction pad stiffening metal sheet mass; m_2 , friction pad mass; $k_{1,x,y}$, longitudinal and lateral stiffness of the brake holder; $c_{1,x,y}$, longitudinal and lateral damping of the brake holder; $k_{2,x,y}$, longitudinal and lateral stiffness of the reinforcing sheet; $c_{2,x,y}$, longitudinal and lateral damping of the reinforcing sheet; k_3 , stiffness of the friction pad; c_3 , damping of the friction pad; N , pressure force of the pad on the brake disc; F_B , inertia.

The studies have analysed only the movement perpendicular to the brake disc, which is why the model shown in Fig. 2a was simplified to the model schematically shown in Fig. 2b. The movement of this system is described by a set of equations.

$$\begin{cases} m_1 \ddot{y}_1 + c_1(\dot{y}_1 - \dot{y}_0) + k_1(y_1 - y_0) + c_2(\dot{y}_1 - \dot{y}_2) + k_2(y_1 - y_2) = N \\ m_2 \ddot{y}_2 + c_2(\dot{y}_2 - \dot{y}_1) + k_2(y_2 - y_1) + c_3(\dot{y}_2 - \dot{y}_0) + k_3(y_2 - y_0) = 0 \end{cases} \quad (2)$$

Recently, a widely discussed model of a brake system is the one presented by Rudolph and Popp [7,8]. In 1972, North [9] developed a special model of a brake system with eight degrees of freedom. Figure 2 shows the dual mass model of the friction pad, pressed with the braking holder to the brake disc with force N .

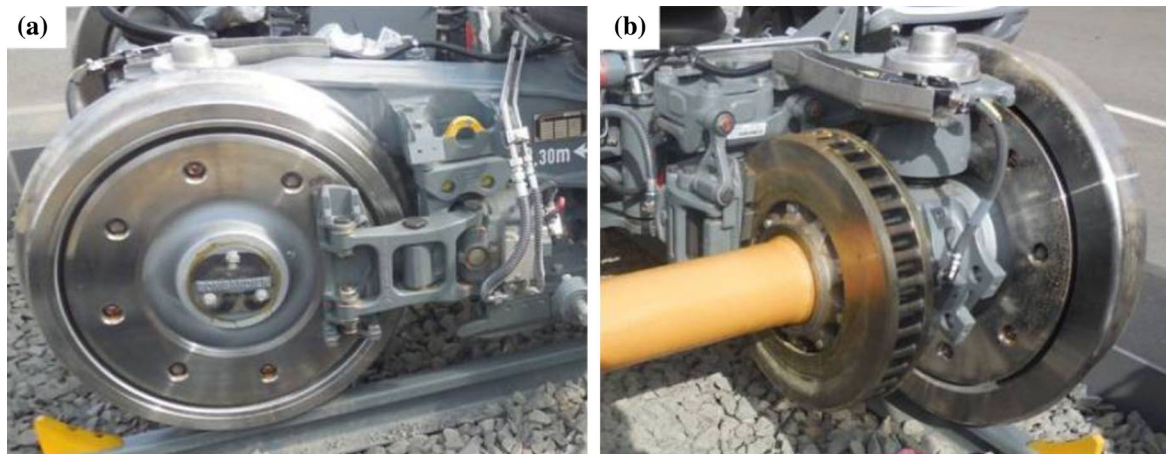


Fig. 1 View of the disc brake friction pair cooperating with the disc mounted: **a** at the wheel of the wheelset, **b** on the axis of the wheelset

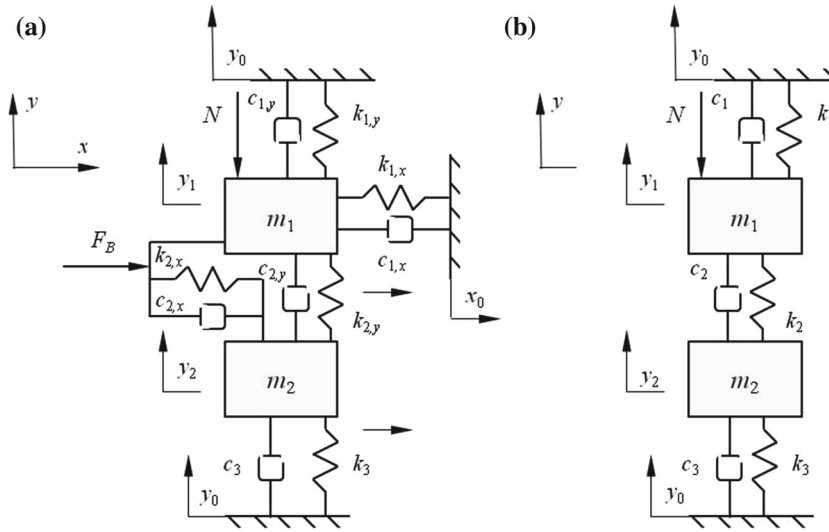


Fig. 2 Dual mass model with the friction of the resilient friction system: **a** the movement of the system in two directions, **b** motion of the system in one direction

First models assumed that the brake self-excited vibrations were associated with a decrease in the coefficient of friction and the increase of the lost motion speed. This is true for many friction materials but for a limited range of speed changes, if we assume a brake model with only one degree of freedom [10].

3 Vibroacoustic diagnostics of the railway disc brake

The evaluation of the technical condition of the equipment utilizes the parameters of the accompanying processes, the operating processes as well as the parameters of the nondestructive testing processes (e.g. ultrasound). The quantities that describe the operating processes used in the diagnostics are: power, torque, reactive instantaneous angular velocity etc. They are used as generalized parameters of the technical condition.

Parameters of the accompanying processes (vibration, noise, thermal processes, wear, etc.) are used to assess a detailed technical condition of the object and the location of the fault. The most commonly used accompanying processes in diagnostics are the vibroacoustic processes (noise and vibration).

In [11] Sawczuk and Szymanski described the possibility of applying selected point measures characterizing the vibratory signals for the estimation of the friction disc brake technical condition. The authors conducted their study on an inertia test stand allowing a simulation of the actual braking conditions according to the specified program.

The authors of [12] used a time series analysis of vibration signals to determine the deceleration time, which they compared with the time measured by the inertia test stand equipment.

When analysing the time courses for the given pad thicknesses, self-excited vibrations were observed for the pads worn to 15 mm. The phenomenon of self-excited vibrations may be associated with the change of the system dynamic properties caused by the change of the liner mass, which is clearly visible at the end of the braking process.

In [13] the author verified the regression diagnostic models determined based on the laboratory and field tests. Based on the analysis of the performed research, the authors have found that it is possible to diagnose the wear of the friction pads of a brake disc by utilizing selected parameters that describe the process of friction and the vibration signal generated by the holder of the friction pad. The authors have developed algorithms to diagnose the friction pair of the disc brake.

Upon the analysis of the literature, the authors have found that there are no scientific works treating on the assessment of the disc brake friction pad thickness based on the analysis of the time-frequency vibration signals.

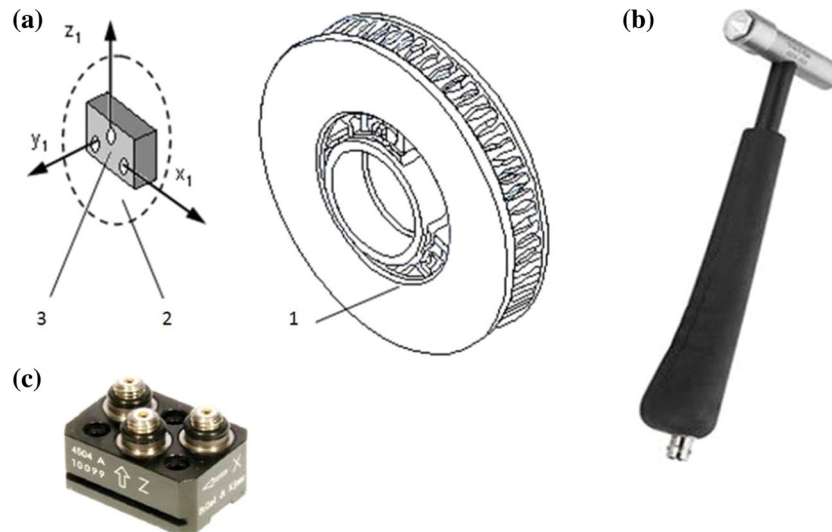


Fig. 3 Pulse test: **a** the direction orientation of the signal recording, **b** view of the 8206-002 modal hammer, **c** view of the 4504 transducer, 1 brake disc, 2 transducer to disc mounting element, 3 vibration transducer

4 Methodology of research

Studies of the self-vibration frequency of the railway brake components were based on the active experiment assumptions [14]. The experiment involved an active intentional change or distortion of the input parameters and the observation of the effects of these changes on the output parameters. For the input parameter, a excitation force applied in the pulse test was adopted, while for the output parameters—the vibrations acceleration of selected components of the disc brake lever mechanism.

Triaxial vibration transducers (type 4504 of Brüel&Kjær) were used for the tests. The linear frequency response of the selected transducers was 18 kHz. During the tests, signals in the range of 0.1 Hz–18 kHz were recorded. The sampling rate was set at 36,000 Hz, which means that the band under analysis, according to the Nyquist relationship, may reach 18 kHz.

The vibration transducers were fixed to a disc brake system with beeswax. Such a method of fitting of the transmitters does not narrow the band of analysis resulting from the characteristics of the transmitter. Directions of the vibration measurement were realized as follows: direction X parallel to the plane of the tested component (in the direction of the axis of rotation), direction Z parallel to the plane (direction of rotation) and direction Y perpendicular to the surface of the tested component (Fig. 3). The pulse was generated with a 8206-002 modal hammer by Brüel&Kjær shown in Fig. 3b.

The PULSE multianalyzer by Brüel&Kjær was used to record the vibration signals. The device allows recording fast-varying electrical waveforms simultaneously on 17 channels with the dynamics of 160dB. The view of the PULSE multianalyzer on the test stand in the Laboratory of Vehicle Design and Diagnostics of Poznan University of Technology has been shown in Fig. 4a. The tests were conducted for the following components and units of the railway disc brake: ventilated brake disc (type 640×110), disc brake lever mechanism, brake pad and brake holder with the pads.

A brake disc was hung on a linear sling to a chain winch by the hole of the wheel seat. The centrifugal force was applied to the friction ring throughout its width from a vibration transducer mounted on it and on the friction ring behind the ventilation part. The view of the brake disc with the fitted vibration transducers during the pulse tests has been shown in Fig. 4c.

Figure 5 shows the view of the clamping mechanism of the disc brake during the pulse test. The vibration transducers were mounted in the middle of the brake holders as in the test stand and validation procedures. When selecting the measurement point, an assumption was made that the transmitter should be placed as close to the source of the vibration signal as possible. In the first case, it hits the brake holder at the parallel guides. In the second case, centrally (vibration transducers fitting spot), while in the third case, at the bottom of the brake holder. The next stage of the study was the measurements of the friction pads at different wear levels. Figure 6 shows the view of the friction pads of a railway disc brake during the pulse test, and Fig. 7 shows the view of the disc brake clamp with the friction pads.

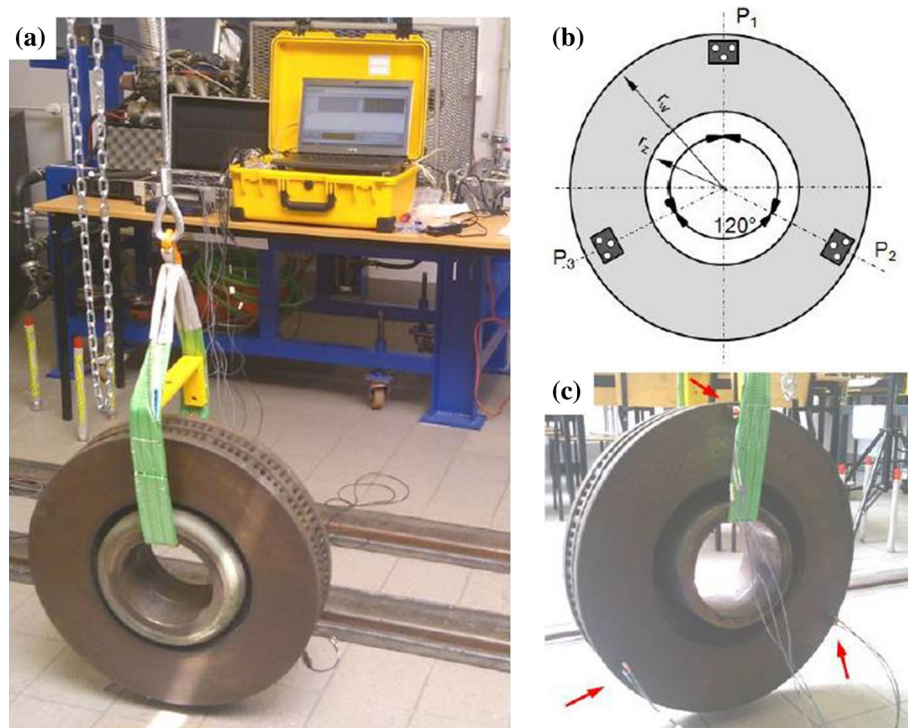


Fig. 4 Pulse test on the brake disc: **a** view of the PULSE multianalyzer on the test stand with the object of the research, **b** fitting spots of the vibration transducers on the disc, **c** view of the brake disc during the pulse test with the vibration transducers, r_w —inner radius of the disc, r_z —outer radius of the disc, $P_{1,2,3}$ —vibration transducers

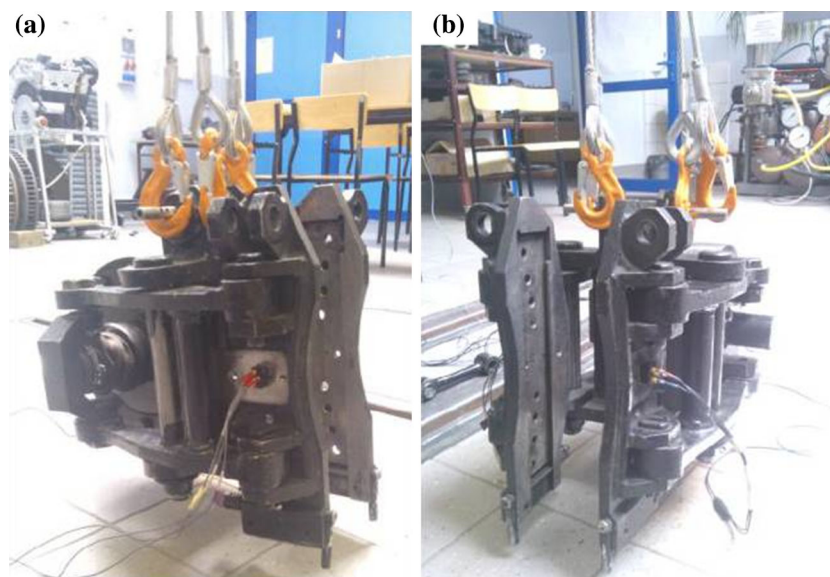


Fig. 5 View of the disc brake clamp without the pads: **a** from the side of the piston rod, **b** from the brake cylinder housing

The clamp (disc brake lever system) without the pads was mounted to a linear sling ended with hooks. In order to eliminate the influence of the sling on determining of the natural frequency of the clamp self-vibrations, a thin tube was inserted in the middle catch, located on the central lever.

A more accurate method of predicting the vibrations of the brake system is the finite elements method. Increased complexity of the models, however, did not help understand the mechanism of the vibration in the

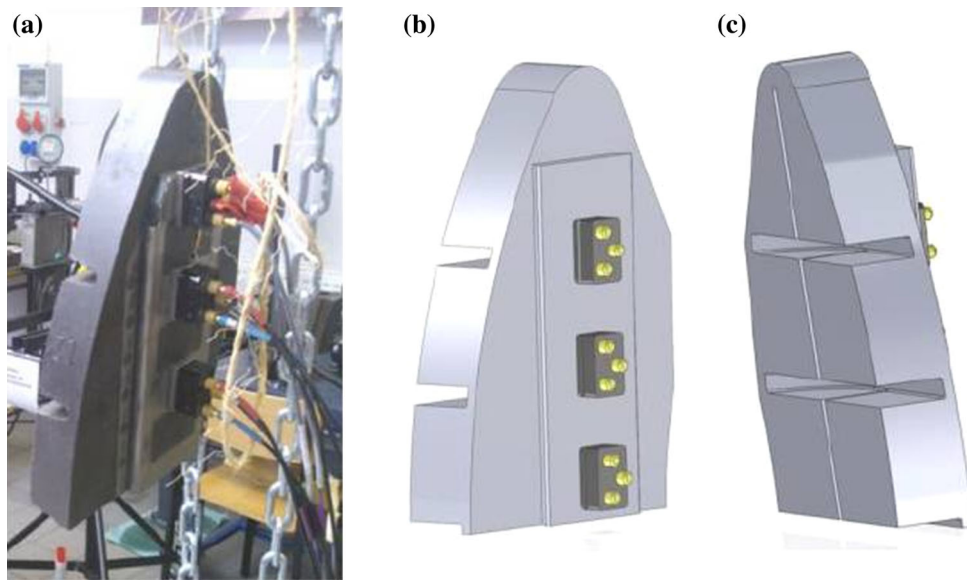


Fig. 6 Friction pad during the pulse test: **a** view of the pad with the vibration transducers, **b** a model of the pad with the vibration transducers, **c** a model of the pad with visible grooves

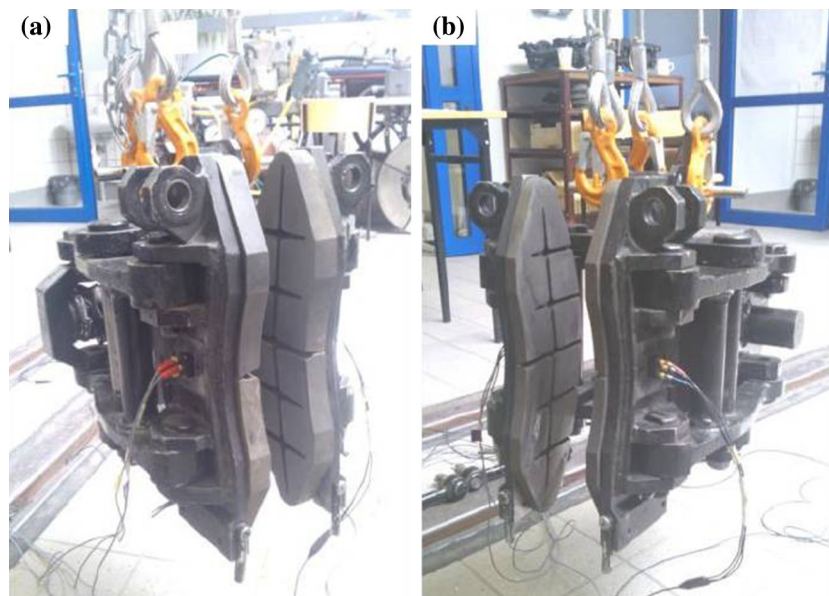


Fig. 7 View of the disc brake lever mechanism: **a** with new pads of the thickness of 35 mm, **b** with the pads worn to 15 mm

brakes. The reason for this is the confining of the modelling based on the finite elements to insulated elements rather than interconnected systems.

An actual operation research was carried out during the driving tests of the railway coach on the route Poznan-Rzepin. The object of the study was a brake system of a 136AMg passenger coach equipped with a MD 523 bogie. The view of the MD 523 bogie with the vibrations transducer attached to a lever of the brake disc is shown in Fig. 8.

The test equipment for the acquisition of the vibration signals was composed of:

- Piezoelectric vibration transducers B & K 4504, 3 pcs.,
- Cassette measuring B&K, 3560 C,
- PULSE system software.

The schematics of the measurement track of the vibrations on the MD 523 bogie is shown in Fig. 9.

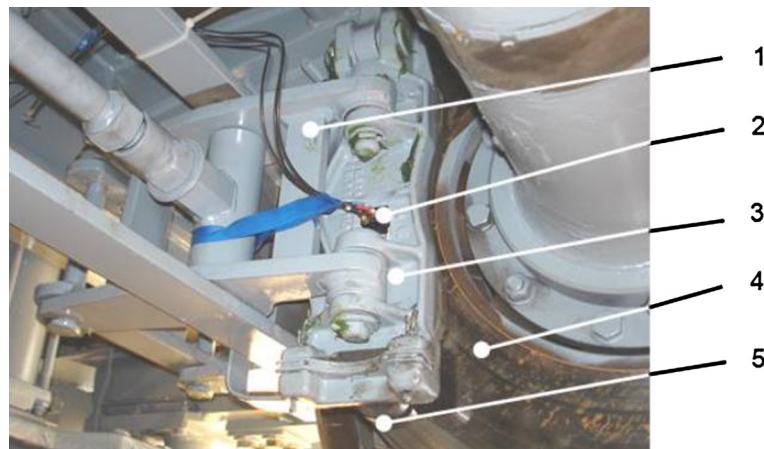


Fig. 8 View of the fitting spots of the vibration transducers to the lever of the Minden-Deutz MD 523 bogie brake disc 136AMg: 1 lever mechanism of the disc brake, 2 vibration transducer, 3 pad holder (caliper), 4 disc brake, 5 friction pad

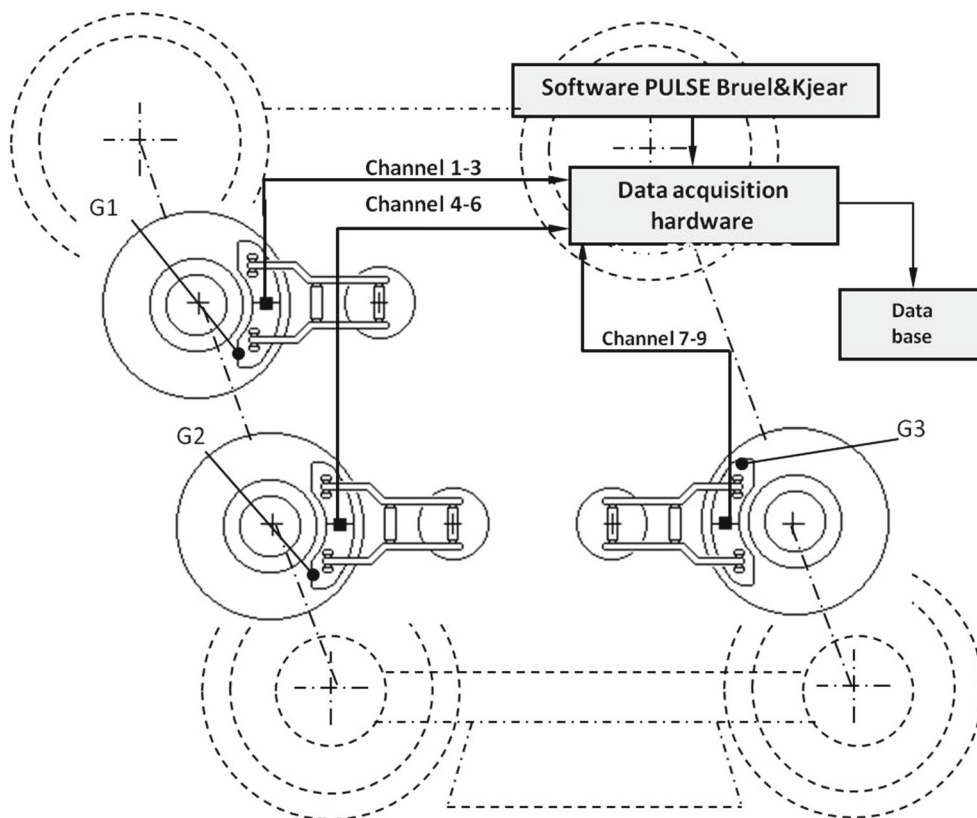


Fig. 9 Measurement track of the friction pads vibrations on the Minden-Deutz MD 523 bogie of the 136AMg passenger coach

4504 Brüel&Kjær vibration transducers were used for the tests. The linear frequency response of the transducers was 13 kHz. During the diagnostic tests, signals were recorded in the range from 0.1 Hz to 10 kHz. For the analysis of the results, significant changes in the vibration signals were observed to approximately 8 kHz. The sampling frequency was 32 kHz, which denotes that the band under analysis (according to the Nyquist relationship) was 16 kHz.

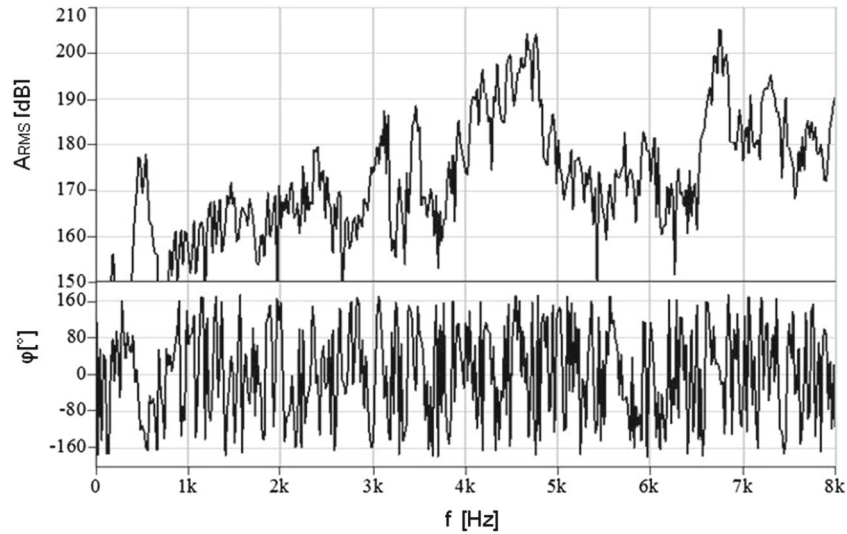


Fig. 10 Amplitude-frequency (*upper* part of the figures) and phase-frequency (*bottom* part of the figures) characteristics following the pulse test carried out on the brake system components (clamp with the pads)

5 Resonance frequencies of the tested brake parts

In the suggested method of diagnostics of the brake components, it is necessary to estimate the resonance frequencies of the brake components that are subject to the assessment of their technical condition.

It is known from the vibration theory that the resonance frequency is associated with the increase of the band amplitude and the change of the sign of phase in the phase band, which is why, in order to define the value of the resonance frequency of the brake system components, a pulse test with the use of external force (modal hammer) was applied. The use of such a test enables defining the frequency response function described by Eq. (3). From the complex form, the amplitude-frequency characteristics (which is a module of the complex frequency) and phase-frequency characteristics are determined.

To identify the dynamic parameters of the structure and the resonance frequencies, elements of modal analysis i.e. pulse tests, were applied. In the case of a classic modal analysis, the external source of the stimulating power (modal hammer or an activator) is applied. Such a solution ensures a simultaneous measurement of the force and the response of the unit to the applied force and enables determining the frequency response function defined by the following equation:

$$H(j\omega) = \frac{X(j\omega)}{F(j\omega)} \quad (3)$$

where $X(j\omega)$ is the complex spectrum of the unit response; $F(j\omega)$ is the complex spectrum of the force stimulating the vibration of the unit; $H(j\omega)$ is the complex function of the frequency response.

In the case of objects for which an external source of power generating the vibrations cannot be applied, there is a possibility of application of an experimental analysis based on internal forces referred to as operating modal analysis. In order to determine the characteristic bands of the signal frequencies bound with the resonance vibrations of the brake parts, pulse tests were carried out according to the assumptions presented in [15]. The sample amplitude-frequency characteristics obtained in the pulse test has been presented in Fig. 10.

Based on the obtained characteristics, the frequencies of own vibrations of the brake system components have been shown in Table 1.

The experimentally determined vibration frequencies of the discussed brake system components will be used to create regressive and structural diagnostic models that enable the assessment of the material consumption in the parts of the brake system using the analysis of vibration signals recorded on the brake pad holder. In order to develop the described models, it would be best to apply those frequencies that occur for each element. In the present case, it is a band of 4.4–4.6 kHz (Table 1).

Another method of determining of the natural frequency of the resonant system is the analysis of the time-frequency map. The analysed vibration signals were recorded in three mutually perpendicular directions for

Table 1 Estimated values of the resonant vibration frequency of the components/units of the railway disc brake

Component/unit	Frequency (kHz)									
Disc	3.0		4.0	4.2	4.5	4.7		5.8	6.2	6.9
Clamp without pads			4.0		4.5			5.9	6.2	7.7
Brake pad					4.5	4.7	5.1	5.8		
Clamp with pads	3.0	3.5		4.2	4.5	4.7			6.7	7.7

Bold values indicate the common frequency band vibrations of selected components or assemblies

different thickness values of the friction pads. The orientation of the measurement direction of the vibrations with respect to the brake disc is shown in Fig. 3.

The signals recorded during the process of braking from 60 to 40 km/h were subject to analysis. The time of the vibration signals is shown in Fig. 11.

The time-frequency analysis was performed with the short-time Fourier transform (STFT). It consists in analysing of the FFT short signal sequence in which it can be regarded as quasi-stationary. The results of the analyses are presented in the form of a time - frequency map in Fig. 12.

Based on the analysis of the time-frequency maps of the signals, it was found that when assessing the thickness of the pads, the analysis of the vibration signals from 6.5 to 8 kHz can be applied (band resonance clamp with blocks Table 1).

6 Algorithm of the assessment of the condition of the brake system components

The algorithm with a binary model is presented in Fig. 13. In the presented algorithm, the procedure of the band pass filtering has been used. The band analysis is set up individually for each tested brake system—by carrying out the pulse tests (the resonance band of the components of the brake system with the largest energy share is applied).

According to the presented algorithm, for the two-phase assessment of the components of the brake system technical condition, known characteristics (discriminants) describing the vibration signal could be used.

7 Application of the proposed method for the assessment of the thickness of the brake pads

One of the parameters that describe the condition of the railway brake disc friction pair is the wear rate of the brake pads manifested by the thickness of the friction pad.

In order to assess the suitability of the proposed method for the assessment of the thickness of the brake pad, an experiment was performed on the inertia test stand used for testing of friction pairs in railway brakes. It has been described in [16]. It consists in determining of the characteristics of the vibration signals for different brake pads (new—G1 = 35 mm used and fit—G2 = 25 mm; used and unfit—G3 = 15 mm).

To boost the resonance vibrations, a pulse was applied in the lever brake system, during the brake hold-off process (separation of the pads from the brake disc).

The obtained calculations of the frequency-amplitude characteristics, are shown in Fig. 14. On the frequency presented in Fig. 14 a, frequency range determined during the pulse test was hatched. The effective value of the vibration signals for different variants of the brake pad wear (G1, G2, G3) was determined (4). Level of acceleration vibration can be described Eq. (5).

$$a_{\text{RMS}} = \sqrt{\frac{1}{T} \int_0^T (a(t))^2 dt} \quad (4)$$

$$A_{\text{RMS}} = 20 \log \left(\frac{a_{\text{RMS}}}{10^{-6}} \right) \quad (5)$$

where $a(t)$, instantaneous value of vibration accelerations in m/s^2 ; a_{RMS} , RMS amplitude in m/s^2 ; A_{RMS} , level of RMS value in dB; t , time; T , average time.

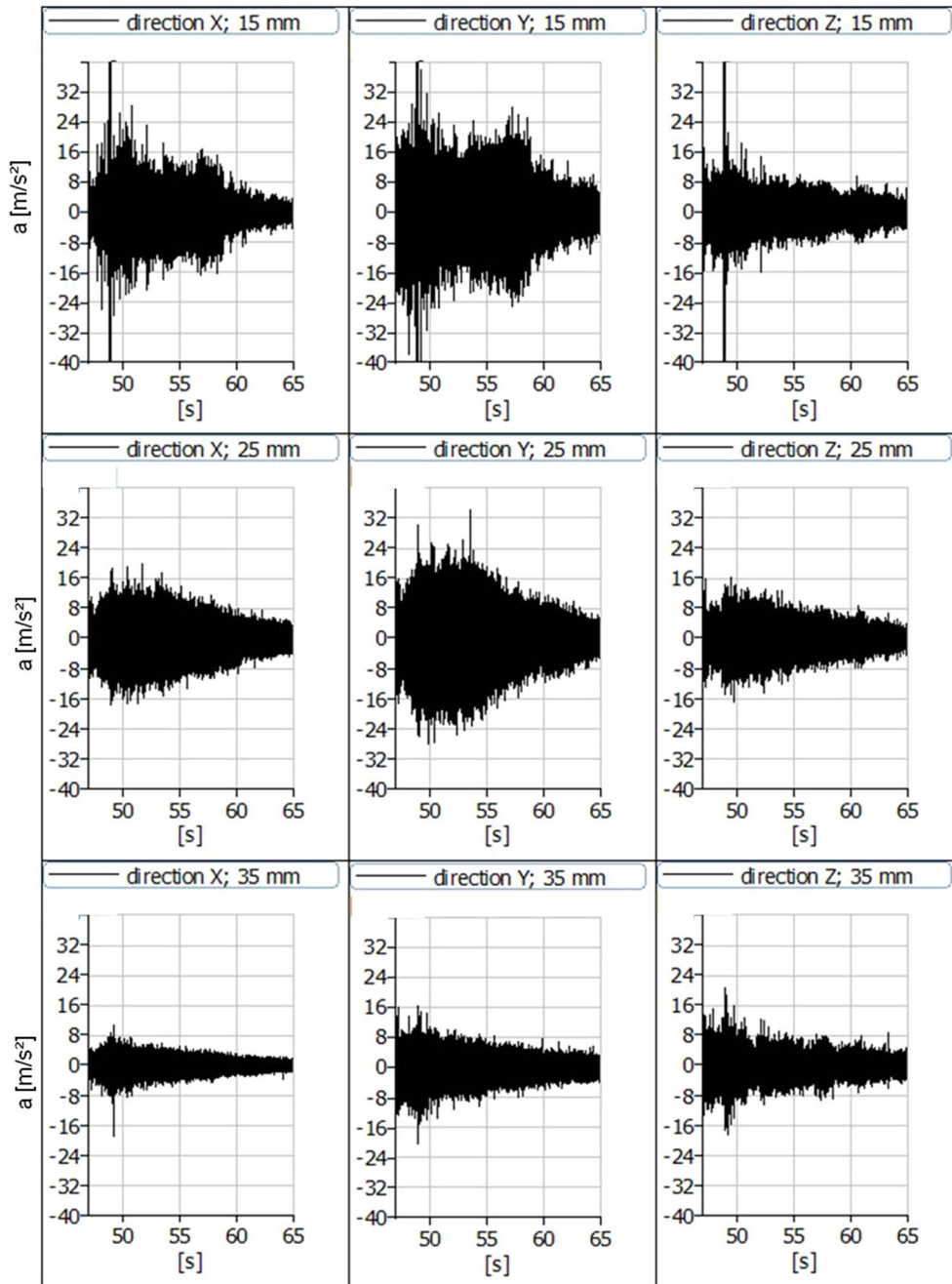


Fig. 11 Time history of the vibrations acceleration signal of the braking pad holders when braking from the speed of 60–40 km/h

The results of the calculations of the measurement point for signals without the use of filtering are shown in Fig. 15a and the effective values of the filtered signals in the band 4.4–4.6 kHz in Fig. 15b.

The dynamics of the diagnostic parameter defined by relation:

$$D = 20 \log \left(\frac{a}{a_0} \right) \quad (6)$$

where D , the rate of the signal change (dB); a , the value of the measured diagnostic parameter for the greatest use of the object; a_0 , the value of the measured diagnostic parameter for the new object.

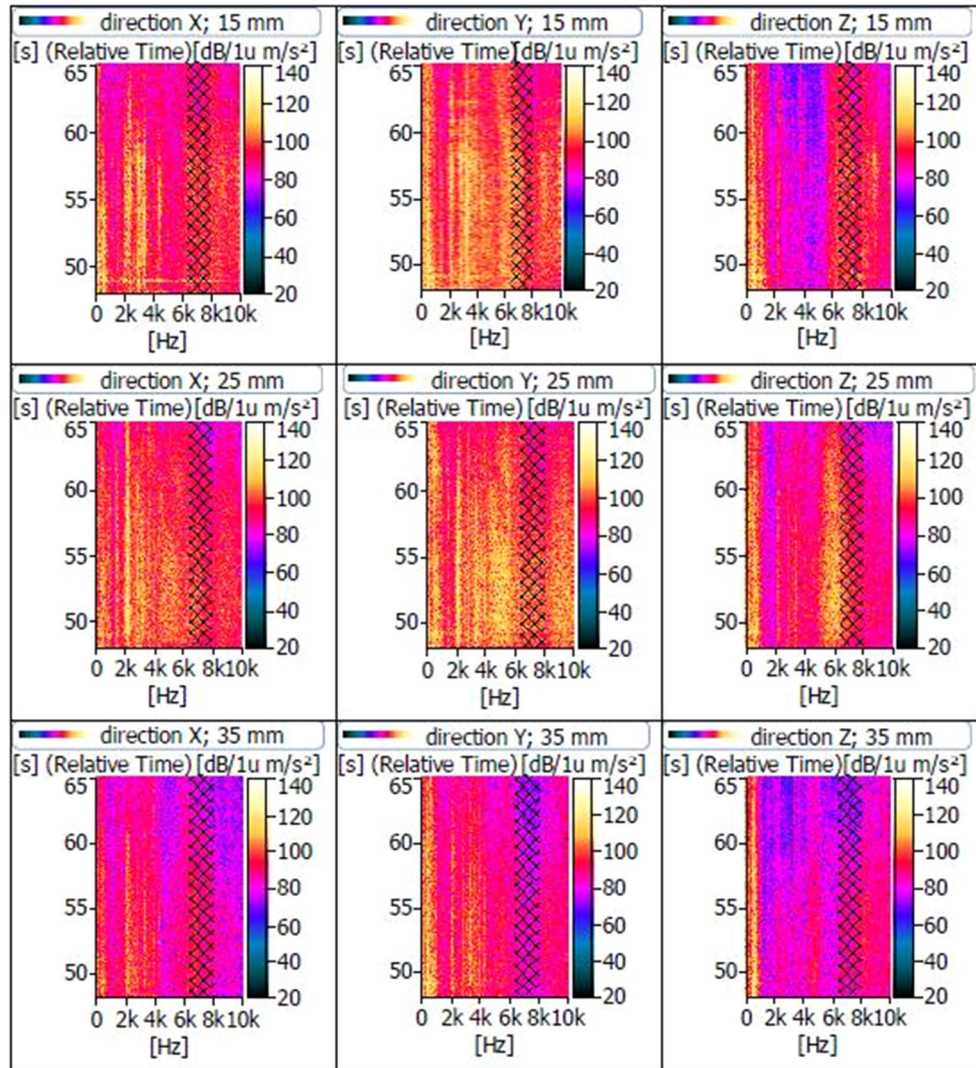


Fig. 12 Time-frequency maps of the vibration acceleration signals of the braking pad holders when braking from the speed of 60–40 km/h

Relationship (6) shows that the dynamics of 6 dB denotes a twofold increase in the diagnostic parameter. In machinery diagnostics, it is assumed that a double increase/decrease of the diagnostic parameter indicates a change in the technical conditions of the object.

During the analysis, the dynamics of the diagnostic parameter was determined with respect to the parameter calculated for a new pad. The results of the calculations for broadband signals are shown in Fig. 16a and for the filtered signals in the resonance band in Fig. 16b.

Based on the analysis of Figs. 14, 15 and 16, it was found that the parameters of the filtered vibration signals in the band 4.4–4.6 kHz can be used for assessing the thickness of the brake pad, as they meet the conditions required of the diagnostic parameters, i.e. they exhibit monotonicity of the waveform as a function of component operation (lack of local extremes), and are easy to measure, and the rate of the signals change is greater than 6 dB.

In order to determine the thickness of the brake pads depending on the parameters that describe the vibratory signals, one needs to set the frequency map parameterization. To estimate the thickness of the brake pad, the maximum value of the vibration in the band of 6.5–8 kHz was applied. The results of the calculations are shown in Fig. 17.

The calculation results shown in Fig. 17 allow the assessment of the thickness of the brake pad. The determined diagnostic parameters are characterized by high dynamics in the range of 20–30 dB (depending

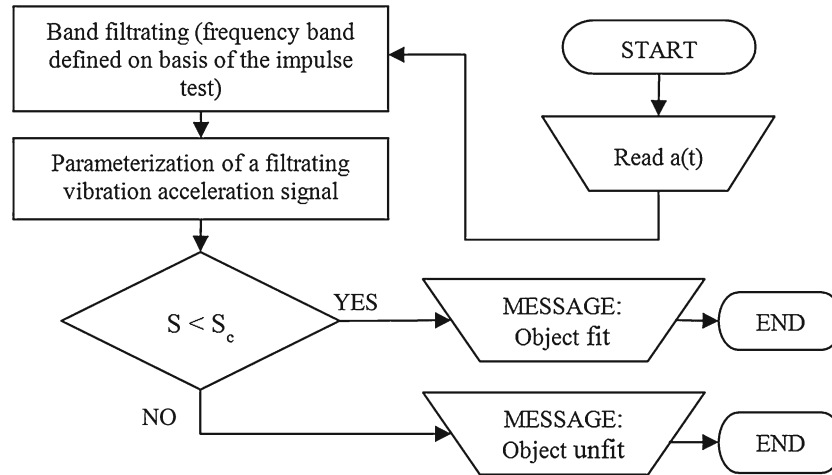


Fig. 13 Algorithm for diagnosing the components of the brake system based on the vibration signal parameters with the use of a binary model ($a(t)$ —the signal of vibration accelerations, S —the diagnostic symptom, S_c —the critical value of the diagnostic symptom)

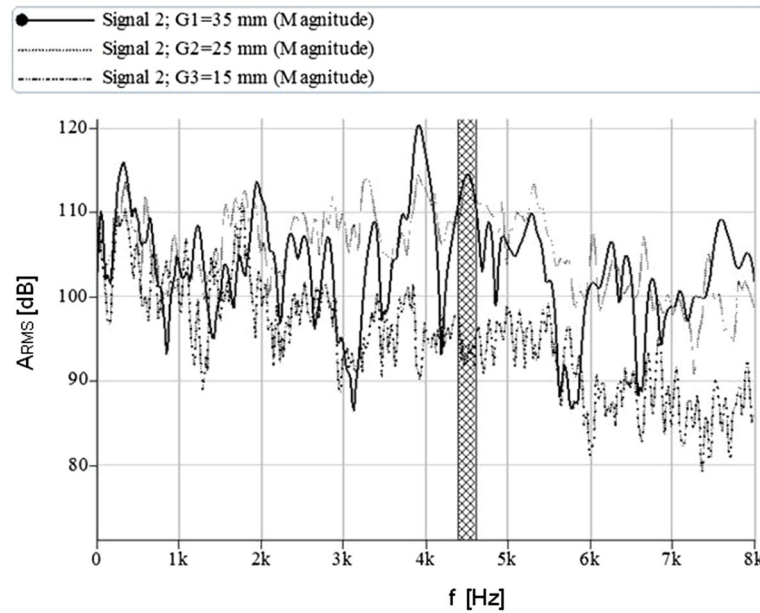


Fig. 14 Frequency characteristics of the acceleration signals recorded on the brake clamp with different thickness values of the friction pads

on the direction of the vibration measurement). The high value of the dynamics of the diagnostic parameter reduces the risk of misdiagnosis.

For the design case, the regression equation was determined.

$$Th = -0.97A_x + 137 \quad (7)$$

$$Th = -0.66A_y + 103 \quad (8)$$

$$Th = -0.62A_z + 99 \quad (9)$$

where Th is the thickness of the friction lining, and A_x , A_y , A_z is the level of vibration acceleration in the band 6.5–8 kHz in the X , Y , Z directions.

After analysing the significance of the regression coefficients and Eqs. (7–9), it was found that the coefficient of determination R^2 for all cases exceeds 0.96, Eq. (8) can be used to estimate the thickness of the friction pad

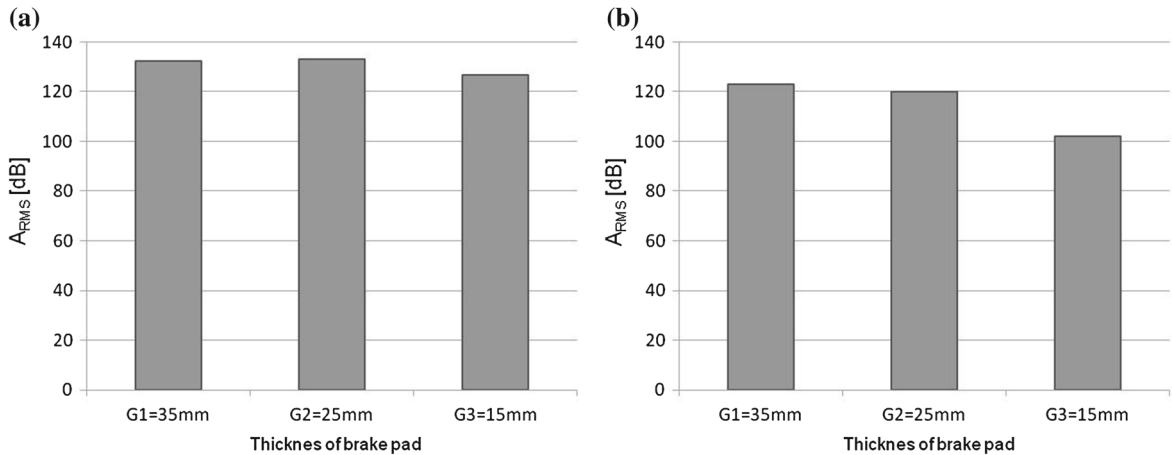


Fig. 15 Calculated effective values of vibration acceleration signals: **a** without filtering, **b** with filtering in the resonance band

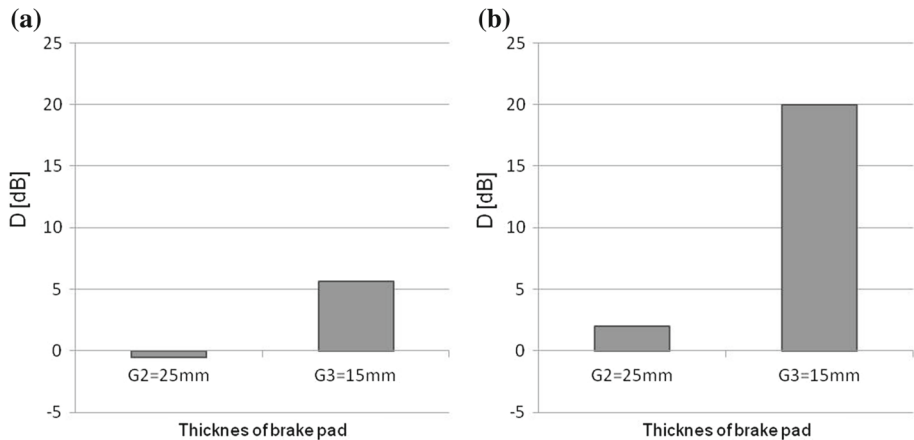


Fig. 16 Calculated values of the dynamic vibration acceleration signals: **a** without filtering, **b** with filtering in the resonance band

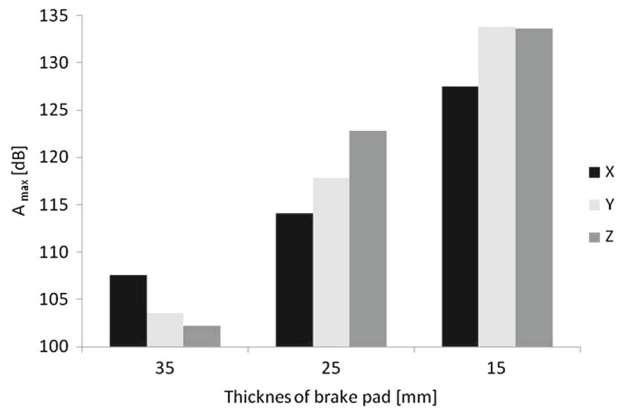


Fig. 17 Dependence of the maximum value of the time-frequency vibration acceleration (6.5–8 kHz band) on the brake pad thickness

and Eqs. (7) and (9) should be rejected because the regression coefficients determined by these equations are statistically insignificant.

In the present case, the statistical regression analysis is indicative because the determined regression coefficients are subject to significant errors resulting from the small sample size (3 points).

8 Conclusions

The paper presented a method of evaluation of the brake pad conditions based on the resonance vibrations measured on the brake block holder. Based on the research, it was found that the vibration signal filtration in the resonance band increases the dynamics of the diagnostic parameter by approximately 14 dB (from 5.6 to 20 dB). Taking a criterion into account, in which it is assumed that the change of the object's technical condition is indicated by the increase of a diagnostic symptom by 6 dB, it can be concluded that the diagnostic model without the use of signal filtration does not guarantee the distinction of the technical condition of the friction components in the tested disc brake, while the signal analysis with the filtration in the band 4.4–4.6 kHz enables a clear evaluation of the technical condition of the tested components. The disadvantage of the described method is that the pulse test should be performed for each brake type in order to determine the resonant frequencies of the tested systems.

The paper also presented the possibility of using the vibration signal to assess the condition of the friction disc brake. After analysing the time-frequency vibration acceleration signals, it was found that it is possible to determine the wear of the friction pads based on the maximum value from the time-frequency map of vibration acceleration in the band of 6.5–8 kHz under actual on-track operation. The best direction in the discussed case is the *Y* direction of the measurement of the vibration acceleration, which is perpendicular to the surface of the friction disc. The determined diagnostic parameters for the signals recorded in this direction are characterized by the dynamics of approx. 30 dB.

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